Technical Notes
on the AICPA Audit Guide
Audit Sampling

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PREFACE

This paper contains technical notes on the 2012 edition of the AICPA Audit Guide *Audit Sampling*. It updates the technical notes on the 2008 edition of the guide. Because there are no changes in the guide's statistical tables these notes are substantially unchanged from 2008. References to the guide have been updated where necessary, and there are a number of minor expositional and other improvements.

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PREFACE TO THE 2008 EDITION

This paper contains technical notes on the 2008 edition of the AICPA Audit Guide *Audit Sampling*. I have written the paper to document the key statistical tables in the guide for the benefit of statistical specialists, educators, students, and others. It will help firms extend the tables to cover their specific policies and guidance, individual practitioners tailor their sampling techniques to specific audit circumstances, and developers write software to augment or replace tables. While I have provided some theoretical background, I have assumed that the reader is familiar with the basics of audit sampling and have focused on the application of theory to the tables.

In the interest of clarity and with practical computation in mind, I have explained matters in terms of functions that are available in Microsoft Office Excel (2007 and 2003 versions), software that is widely used by auditors. These functions can be readily translated into their equivalent in other software, and the use of Excel for the purposes of this paper is not an endorsement of that product or its suitability for statistical calculations.

Section 1, "Definitions and Conventions," defines the terms and conventions used in the paper. Section 2, "Theory and Algorithms," provides enough theory to anchor the discussion in established statistical terms and explains specific formulas and algorithms. Section 3, "Statistical Functions in Excel," shows how the statistical functions required for the tables may be implemented in Excel. Section 4, "Computation of Tables with Examples," shows how each key table can be computed, referring back to the preceding material.

I wish to acknowledge the help I have received from fellow task force members, especially Abe Akresh and Bill Felix.

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I would also like to acknowledge the assistance obtained from several others including Lucas Hoogduin of KPMG LLP and Paul van Batenburg of Deloitte. Further, I thank Donald Roberts of the University of Illinois at Urbana-Champaign for his review of this document.

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1 DEFINITIONS AND CONVENTIONS

Symbols and terms used in this paper are set forth in the following table.

Term	Definition
Error	Deviation or misstatement.
T	Tolerable error.
α	Risk of incorrect rejection: the risk of concluding that the total error or the error rate exceeds what is tolerable when it does not.
β	Risk of incorrect acceptance: the risk of concluding that the total error or error rate is tolerable when it is not.
M	Monetary total of the population.
N	Number of items in the population.
L	Number of errors in the population; also L_T for the tolerable number of errors.
n	Sample size.
p	Population error rate; also, p_E, p_T, and p_U for expected, tolerable, and upper limit rates, respectively.
k	Number of errors in sample; also, k_E for the expected number of errors; may also denote sum of the error taints in monetary unit sampling applications.
r	Mean number of errors in samples of size n; also, r_T and r_U for tolerable and upper limit mean number of errors, respectively.
A	Precision of a classical variables sampling estimate.
σ_n	Standard deviation of an estimator for sample of size n. In the statistical literature, σ_n is often called the *standard error* of the estimator, but in auditing such use of the term *error* can be confusing.
z_ε	Standard normal deviate such that the probability is $1 - \varepsilon$ that $z \le z_\varepsilon$.
MUS	Monetary unit sampling.
RoundUp(x, δ)	The variable x rounded up to the nearest δ decimal places. When $\delta = 0$, x is rounded up to the nearest integer.
exp(x)	The constant e (Euler's $e = 2.7182\ldots$) raised to the power of x. For $x > 0$, it is the inverse (the antilogarithm) of $\ln(x)$, thus $\exp(\ln(x)) = x$.
ln(x)	The natural logarithm (base e) of x. This is the inverse of $\exp(x)$; thus $\ln(\exp(x)) = x$.
VBA	Microsoft Visual Basic for Applications, as implemented in Excel 2007 and 2003.

The following mathematical conventions are used for probability distributions. For discrete probability distributions, namely the hypergeometric, binomial, and Poisson distributions, the *probability mass function* is the probability of obtaining exactly k errors. These are denoted as $Hyp(k,\ldots)$, $Bin(k,\ldots)$, and $Poi(k,\ldots)$. The *cumulative distribution function* is the probability of obtaining k or fewer errors and is the sum of the mass function from zero to k. Cumulative distribution functions are denoted by prefixing the letter C to the symbol for the mass function; thus $CHyp(k,\ldots)$, $CBin(k,\ldots)$, and $CPoi(k,\ldots)$.

For continuous probability distributions, the *probability density function* at point x is the ordinate of the function at that point. The density function defines the probability curve (for example, the familiar bell-

shaped curve for normal distributions). The cumulative distribution function is the probability that total error or error rate, depending on the context, does not exceed x and is represented by the area under the curve to the left of x. Mathematically, it is the integral of the density function to the left of x. The beta and gamma density functions are denoted by $b(x,\ldots)$ and $g(x,\ldots)$, respectively, while their cumulative probability distributions are denoted by $B(x,\ldots)$ and $G(x,\ldots)$. The cumulative normal distribution is denoted by $N(x,\ldots)$. Inverse distributions (also known as percentile functions) express the value of x for a given probability u. They are denoted by $B^{-1}(u,\ldots)$, $G^{-1}(u,\ldots)$, and $N^{-1}(u,\ldots)$ for the inverse beta, gamma, and normal distributions, respectively.

2 THEORY AND ALGORITHMS

This section describes the theory underlying the tables in the Audit Guide *Audit Sampling* (the guide) and the various formulas and algorithms that can be used to compute them. Because most of the tables relate to attributes sampling and MUS, that is the focus of this section—except for section 2.5, which deals with the relation between precision and tolerable misstatement in classical variables sampling.

In attributes sampling, we have a population of N items that contains L errors. A sample of n items is randomly selected from the population and includes k errors. The sample design problem is to determine how large n should be so that if the audit goes as expected the auditor will be able to conclude with some appropriate degree of confidence (for example, 95%) that the number of errors or error rate does not exceed a tolerable level. The sample evaluation problem is to determine, based on the number of errors detected in the sample, an upper limit such that the auditor has an appropriate level of confidence that the actual number of errors or error rate does not exceed that upper limit.

In attributes sampling and MUS, the guide focuses on directly specifying the risk of incorrect acceptance β. The risk of incorrect rejection α is not specified directly but is influenced by specifying the expected number of errors or error rate. The larger this is, the larger the resulting sample size and the lower the risk of incorrect rejection.

2.1 The Hypergeometric Probability Distribution

The hypergeometric probability mass function expresses the probability that k errors will be selected in a sample of n items randomly selected without replacement from a population of N items that contains L errors. Its derivation is described in the next paragraph.

The total number of possible samples of n items that can be selected from a population of N items is

$$\binom{N}{n} = \frac{N!}{n!(N-n)!},$$ (1)

where, for example, $n!$ (n factorial) is $n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1$. The number of ways in which k errors can be selected from L errors in a population is

$$\binom{L}{k}.$$

Similarly, the number of ways in which $n-k$ non-errors can be selected from the $N-L$ non-errors in a population is

$$\binom{N-L}{n-k}.$$

Therefore, the total number of ways in which a sample containing k errors and $n-k$ non-errors can be selected is

$$\binom{L}{k}\binom{N-L}{n-k}.$$

This divided by (1), the total number of possible samples of size n, is the probability that the sample will contain exactly k errors and defines the hypergeometric probability mass function:

$$Hyp(k,n,L,N) = \frac{\binom{L}{k}\binom{N-L}{n-k}}{\binom{N}{n}} \tag{2}$$

where $k, n, L, N = 0, 1, 2, \ldots;$ $L \le N;$ $k \le L;$ and $n - (N - L) \le k \le n \le N$

(Johnson, Kemp, and Kotz 2005, 251). For example, the probability of obtaining $k = 2$ errors in a sample of $n = 5$ items selected from a population of $N = 50$ items in which there are $L = 10$ errors is

$$Hyp(2, 5, 10, 50) = \frac{\binom{10}{2}\binom{50-10}{5-2}}{\binom{50}{5}} = \frac{\dfrac{10!}{2!8!} \times \dfrac{40!}{3!37!}}{\dfrac{50!}{5!45!}} = 0.2098 .$$

It can be shown that the mean and variance are

$$E(k) = np \quad \text{and} \quad \text{var}(k) = np(1-p)\frac{N-n}{N-1},$$

where $p = L / N$.

The cumulative hypergeometric probability that the number of errors in the sample is k or fewer is

$$CHyp(k,n,L,N) = \sum_{i=k_0}^{k} Hyp(i,n,L,N) , \tag{3}$$

where

$$k_0 = \begin{cases} 0, & \text{if } n \le N - L \\ n - (N - L), & \text{if } n > N - L \end{cases}$$

is the minimum number of errors that the sample can possibly contain.[1]

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[1] As pointed out by Feller (1968, 44), the summation in function (3) can start at 0 rather than k_0. It starts with k_0 in function (3) because the Excel HYPGEOMDIST function returns a #NUM! error if $i < k_0$.

2.1.1 Attributes Sample Sizes Using the Hypergeometric Distribution

Given the risk β, the population size N, the tolerable number of errors in the population L_T, and expected number of errors in the sample k_E, function (3) can be used to compute the required sample size n. It is the minimum value of n for which

$$CHyp\left(k_E, n, L_T, N\right) \leq \beta . \tag{4}$$

The guide assumes that auditors specify a tolerable error rate p_T instead of a tolerable number of errors L_T and an expected error rate p_E instead of an expected number of errors k_E. This causes a minor practical difficulty when, as is generally the case, the error rate does not translate into an integral number of errors. For instance, a population size of 52 and an error rate of 10% implies 5.2 errors. Because, in attributes sampling, an item is either an error or not an error it is necessary to decide whether that means 5 or 6 errors. On the basis that partial errors do not exist, we round up to the nearest integer. Thus 5.2 is rounded up to 6.

While the hypergeometric distribution is the exactly correct distribution to use for attributes sample sizes, it gets unwieldy for large populations unless suitable software is available. It is principally for this reason that more convenient approximations are frequently used in practice.

Example: See section 4.4 of this paper.

2.1.2 Practical Limitations of the Hypergeometric Distribution

The hypergeometric distribution takes into account that audit sampling is typically performed *without replacement*: sample items are effectively extracted from the population and cannot be re-selected. This means that the probability that a sample item will be an error depends on how many errors and non-errors have already been selected and is not constant from selection to selection. This complicates the mathematics and makes it infeasible to prepare comprehensive tables that go beyond quite small populations.

The advent of widely deployed popular software such as Excel has reduced the need for tables, but this is a relatively recent development and does not address analytical complexities. Simpler distributions that approximate the hypergeometric distribution are often used. In audit sampling, the most frequently used approximations, the ones used in the guide, are the binomial and the Poisson probability distributions.

2.2 The Binomial Probability Distribution

Another way to select a sample is *with replacement*: sample items are not extracted from the population but are available for reselection. This makes sampling easier to analyze, describe, and tabulate because the probability that the next sample item will be an error remains constant no matter how large the sample. When a sample is small relative to the population, it makes little difference whether it is selected with or without replacement because the probability of selecting an item more than once is relatively small. In these circumstances, it is usually reasonable to make the simplifying assumption.

The probability that an item selected at random with replacement from the population will be an error is $p = L / N$, the population error rate, and this is constant for each item selected. If n selections are made, each sample item has a probability p of being an error and a probability $1 - p$ of being a non-error. Therefore, the

probability that a sample of n items will be some given combination of k errors and $n - k$ non-errors is $p^k(1-p)^{n-k}$. Because there are

$$\binom{n}{k}$$

such possible combinations, the total probability is the product of these two factors. More formally, the binomial probability mass function expresses the probability that k errors will be selected in a sample of n items randomly selected with replacement from a population in which the population error rate is p. It is

$$Bin(k,n,p) = \binom{n}{k} p^k (1-p)^{n-k} \tag{5}$$

(Johnson, Kemp, and Kotz 2005, 108).

The cumulative probability that the number of errors in the sample is k or fewer is the sum of the mass function (5) from 0 to k, namely

$$CBin(k,n,p) = \sum_{i=0}^{k} Bin(i,n,p). \tag{6}$$

It can be shown that the mean and variance are $E(k) = np$ and $var(k) = np(1-p)$, respectively.

A common rule of thumb is that the binomial can be used in place of the hypergeometric if $n < 0.1N$ (Johnson, Kemp, and Kotz 2005, 269). In any case, as explained in section 2.4 of this paper, the binomial provides a conservative approximation for audit applications in the sense that it results in sample sizes and evaluated upper error limits that are at least as large as those calculated using the hypergeometric distribution. When suitable software is available for hypergeometric calculations, however, there may be little benefit to using the less efficient, though simpler, binomial approximation.

2.2.1 Attributes Sample Sizes Using the Binomial Distribution

Given risk β, tolerable error rate p_T, and expected error rate p_E, the required sample size is the minimum value of n for which

$$CBin(k_E, n, p_T) \le \beta, \tag{7}$$

where $k_E = \text{RoundUp}(np_E, 0)$. Because there is no explicit formula solution, n must be found numerically. A VBA function for doing so,

$$n = \text{BinomSample}(\beta, p_E, p_T), \tag{8}$$

is presented in section 3.2.1 of this paper.

Examples: See sections 4.1–4.3 and 4.10–4.11 of this paper.

2.2.2   *The Beta Distribution and its Relationship to the Binomial*

The binomial distribution has associated with it a continuous probability distribution, the *beta* distribution, which describes the probability that *p*, the population error rate, could be less than or equal to any given value in the range from 0 to 1, given the size of the sample and the number of errors selected.

The beta distribution can be defined mathematically by the probability density function

$$b(p, 1+k, n-k) = (n-k)\binom{n}{k} p^{k}(1-p)^{n-k-1}$$

(9)

$$\text{where} \quad 0 \le p \le 1; \quad n = 0,1,2,\ldots; \quad \text{and} \quad k = 0,1,2,\ldots,n$$

(DeGroot 2002, 303).[2] The cumulative distribution is

Figure 1, The Binomial Distribution and Related Beta Distribution

*Bin(k,*100,0.076): Binomial probability mass function with parameters *n* = 100 and *p* = 0.076. Number of errors (*k*) is measured on the horizontal axis; probabilities are represented by the heights of the bars. The probability that *k* ≤ 3 is 5%.

*b(p,*4,97): Beta probability density function with parameters 1 + *k* = 4 and *n* − *k* = 97. Potential population error rate *p* is measured along the horizontal axis; probabilities are represented by areas under the curve. Based on the results of the sample (*k* = 3), the projected population error rate is 0.03 (where the curve peaks) and its 95% upper confidence limit is 0.076. The vertical axis is scaled such that the total area under the curve is 1.

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[2] The beta distribution *b(p, θ, λ)* is usually defined more generally for *θ, λ* > 0, but it reduces to this formulation for integer values *θ* = 1 + *k* and *λ* = *n* − *k*.

$$B(p,1+k,n-k) = \int_0^p b(t,1+k,n-k)dt; \ 0 \le p \le 1. \tag{10}$$

The principal relevance here of the beta distribution is that the cumulative binomial probability can be written in terms of it,

$$CBin(k,n,p) = 1 - B(p,1+k,n-k) \tag{11}$$

(Abramowitz and Stegun 1972, 945, eq. 26.5.24). This equation simply expresses a mathematical equality that is useful in planning and evaluating attributes and MUS samples.[3] An example of the relationship between the binomial and beta distributions is shown in figure 1.

2.2.3   Calculating the Upper Error Rate Limit

The upper error rate limit can be found using the binomial distribution. It is the value p_U such that

$$CBin(k,n,p_U) = \beta .$$

This value can be found numerically through successive approximations. In practice, though, it is easier to use the beta distribution. From equation (11) it is the value p_U for which $B(p_U,1+k,n-k) = 1 - \beta$. This can be computed directly using the inverse of the beta distribution:

$$p_U = B^{-1}(1-\beta,1+k,n-k). \tag{12}$$

The advantage of this formulation for present purposes is that the inverse beta distribution is provided directly in Excel.

Examples: See sections 4.12–4.13 of this paper.

2.3   The Poisson Probability Distribution

The Poisson is an important probability distribution that arises in many stochastic processes. In sampling, it arises as an approximation to the binomial distribution. Its usefulness lies in its simplicity: sample sizes are relatively easy to calculate, without the need for software or extensive tables in many cases, and sample evaluation is also relatively straightforward. A common rule of thumb is that the Poisson provides a good approximation to the binomial when $n \ge 20$ and $p \le 0.05$ (Freund 1972, 83). For any given $r = np$, the approximation improves as p decreases and n increases.

The Poisson is often used as an alternative to the binomial for audit sampling, especially in MUS. As explained in section 2.4 of this paper, it is a conservative approximation for audit applications in the sense

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[3] A Bayesian interpretation of equation (11) is that the probability of obtaining k or fewer errors in a sample of n items from a population in which the error rate is p equals the probability that the error rate exceeds p given that k errors were detected in a sample of n items.

that it results in sample sizes and evaluated upper error limits that are at least as large as those calculated using the binomial distribution. This loss of efficiency is usually minimal and is often regarded as worthwhile for the resulting greater simplicity.

Mathematically, the Poisson arises as a limiting case of the binomial. The following explanation of this is based on DeGroot and Schervish (2002, 258). For $k \geq 1$ the binomial distribution may be written as

$$\binom{n}{k} p^k (1-p)^{n-k} = \frac{n(n-1)\cdots(n-k+1)}{k!} p^k (1-p)^{n-k} . \tag{13}$$

We now let $n \to \infty$ and $p \to 0$ in such a way that their product np (that is, the expected number of errors in the sample) remains fixed throughout the limiting process. Let $r = np$, then $p = r/n$ and equation (13) can be written as

$$\binom{n}{k} p^k (1-p)^{n-k} = \frac{r^k}{k!} \left(\frac{n}{n} \times \frac{n-1}{n} \times \cdots \times \frac{n-k+1}{n} \right) \left(1 - \frac{r}{n} \right)^n \left(1 - \frac{r}{n} \right)^{-k} .$$

Because the values of r and k are held fixed as $n \to \infty$,

$$\lim_{n \to \infty} \left(\frac{n}{n} \times \frac{n-1}{n} \times \cdots \times \frac{n-k+1}{n} \right) \left(1 - \frac{r}{n} \right)^{-k} = 1 .$$

Furthermore, from elementary calculus,

$$\lim_{n \to \infty} \left(1 - \frac{r}{n} \right)^n = \exp(-r) .$$

The preceding applies for $k \geq 1$, but the same applies for $k = 0$ because

$$\binom{n}{0} p^0 (1-p)^{n-0} = (1-p)^n = \left(1 - \frac{r}{n} \right)^n \to \exp(-r) \text{ as } n \to \infty .$$

It follows, therefore, that for any value of $k = 0,1,2\ldots,n$ and for any $r \geq 0$

$$\binom{n}{k} p^k (1-p)^{n-k} \to \frac{r^k \exp(-r)}{k!}$$

as $n \to \infty$ and $p \to 0$ such that $np = r$. This limiting case of the binomial distribution is the Poisson distribution with mean r. More formally, the Poisson probability mass function

$$Poi(k,r) = \frac{r^k \exp(-r)}{k!}; \quad r \geq 0, \quad k = 0,1,2,\ldots \tag{14}$$

expresses the probability that k errors will be selected in a sample when the mean number of errors in samples of size n is r.

The cumulative probability that the number of errors in the sample is k or fewer is the sum of the mass function from 0 to k, namely

$$CPoi(k,r) = \sum_{i=0}^{k} Poi(i,r) \, . \tag{15}$$

It can be shown that the mean and variance of the Poisson distribution are both r, that is, $E(k) = var(k) = r$.

2.3.1   The Gamma Distribution and its Relationship to the Poisson

The Poisson distribution has associated with it a continuous probability distribution, the *gamma* distribution, which describes the probability that r, the mean number of errors in samples of size n, could be less than or equal to any given value given the number of errors selected.

The standard gamma distribution can be defined mathematically by the probability density function

$$g(r,1+k) = \frac{r^k \exp(-r)}{\Gamma(1+k)}; \quad r \geq 0, \quad k \geq 0, \tag{16}$$

where the gamma function $\Gamma(1+k) = \int_0^\infty t^k \exp(-t)dt$ (Johnson, Kotz, and Balakrishnan 1994, 337).[4] As will be explained, k may be the sum of the error taints for certain purposes and thus a non-integer. If all the errors are 100% taints, however, it is an integer and $\Gamma(1+k) = k!$. The cumulative distribution function is

$$G(r,1+k) = \int_0^r g(t,1+k)dt \, . \tag{17}$$

The principal relevance here of the gamma distribution is that the cumulative Poisson probability can be written in terms of it. For k errors

$$CPoi(k,r) = 1 - G(r,1+k) \tag{18}$$

(Raiffa and Schlaifer 2000, 224). This equation simply expresses a mathematical equality that is useful in planning and evaluating attributes and MUS samples.[5]

An example of the relationship between the Poisson and gamma distributions is shown in figure 2. It may be noted that the charts in figure 2 are quite similar to those in figure 1 except for the fact that the beta

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[4] The gamma distribution $g(x, \theta, \lambda)$ is usually defined more generally for $\theta, \lambda > 0$ where θ is the *shape* parameter and λ the *scale* parameter, but it reduces to this formulation when $\theta = 1 + k$ and $\lambda = 1$. When $\lambda = 1$, the scale parameter disappears and the distribution is known as the *standard* gamma distribution.

[5] A Bayesian interpretation of equation (18) is that the probability of obtaining k or fewer errors if the mean number of errors in samples of size n is r equals the probability that the mean exceeds r given that k errors were detected.

Figure 2, The Poisson Distribution and Related Gamma Distribution
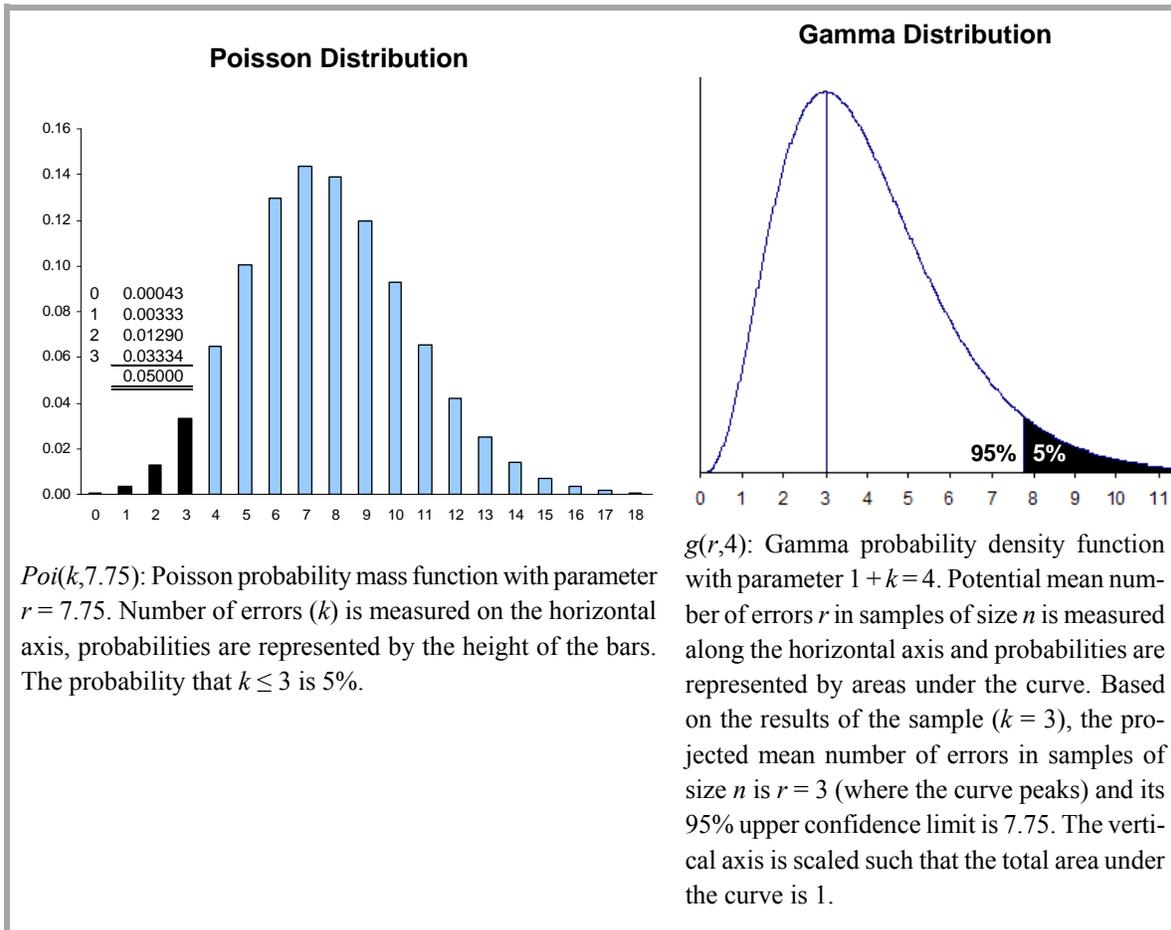
Poi(*k*,7.75): Poisson probability mass function with parameter *r* = 7.75. Number of errors (*k*) is measured on the horizontal axis, probabilities are represented by the height of the bars. The probability that $k \leq 3$ is 5%.

g(*r*,4): Gamma probability density function with parameter $1 + k = 4$. Potential mean number of errors *r* in samples of size *n* is measured along the horizontal axis and probabilities are represented by areas under the curve. Based on the results of the sample (*k* = 3), the projected mean number of errors in samples of size *n* is *r* = 3 (where the curve peaks) and its 95% upper confidence limit is 7.75. The vertical axis is scaled such that the total area under the curve is 1.

distribution is expressed in terms of the potential error rate while the gamma is expressed in terms of the mean number of errors in samples of size *n*.

2.3.2 Evaluating MUS Samples

The upper limit to *r* can be found using the Poisson distribution. It is the value r_U such that

$$CPoi(k, r_U) = \beta .$$

This value can be found numerically through successive approximations. In practice, though, it is easier to use the gamma distribution. From equation (18), the upper limit is the value r_U for which $G(r_U, 1 + k) = 1 - \beta$, provided that the errors are all 100% taints. Using the inverse of the gamma distribution, the upper limit can be computed directly as

$$r_U = G^{-1}(1 - \beta, 1 + k) . \tag{19}$$

The advantage of this formulation for present purposes is that the inverse gamma distribution is provided directly in Excel.

If the errors are not all 100% taints, then k is the sum of the taints and is the best estimate of r. In this situation, the MUS upper limit is typically calculated using the Stringer bound explained in chapter 6 of the guide, which makes use of the increments in the factors calculated in equation (19). However, a good approximation to this upper limit can be obtained by simply using non-integer k equal to the sum of the taints in equation (19). Because errors are ranked in decreasing order of their taints for the Stringer bound and because the incremental factors decline (at least for β less than about 45%), this approximation is conservative in that it results in slightly larger upper bounds than the Stringer bound.

Example: See section 4.16 of this paper.

2.3.3 MUS Sample Sizes

To calculate the appropriate sample size, the auditor establishes an expected population rate p_E (often zero) and determines a sample size n that should be sufficient to result in an upper limit that does not exceed tolerable misstatement. If the expected population error rate is p_E and the sample size is n, it follows that the expected sum of the taints is np_E. If the tolerable error rate is p_T, then $r_U = np_T$. From this and the discussion in section 2.3.2 of this paper, it follows that a conservative approach to estimating the required sample size is to find the minimum value of n for which

$$G(np_T, 1 + np_E) \geq 1 - \beta .$$ (20)

A VBA function for doing this sample size calculation,

$$n = \text{PoissonSample}(\beta, p_E, p_T)$$ (21)

is presented in section 3.2.2 of this paper.

Examples: See sections 4.6 and 4.14 of this paper.

The following alternative approach results in a useful and compact table of factors for MUS design. It hinges on the fact that it is often convenient to express the expected population error rate as a fraction of the tolerable error rate, $e = p_E / p_T$. First, we ignore for the moment that n can only take on integer values and rewrite inequality (20) as an equation expressed in terms of the inverse function:

$$np_T = G^{-1}(1 - \beta, 1 + np_E) .$$

Noting that $np_E = n(ep_T) = e(np_T)$, we can write $F = np_T$ and rewrite the equation as

$$F = G^{-1}(1 - \beta, 1 + eF) .$$ (22)

We can thus generate a table of F factors that depend only on β and e. A VBA function for doing this,

$$F = \text{MUSFactor}(\beta, e),\qquad(23)$$

is presented in section 3.2.3 of this paper. Given the risk β and the ratio e of expected to tolerable error, the sample size is simply

$$n = \frac{F}{p_T}\qquad(24)$$

rounded up; and the expected error is

$$k_E = np_E = eF\ .$$

Example: See section 4.15 of this paper.

2.4 Conservatism of Binomial- and Poisson-Based Sample Design and Evaluation

In auditing, sampling is almost always without replacement, meaning that the hypergeometric distribution is usually the technically correct distribution to use. Some audit software packages support it, in which case there may be no need to use an approximation. Without software support, however, the hypergeometric is awkward to use and the auditor may prefer to use the simpler binomial or Poisson distributions, which usually provide excellent approximations. The binomial is often used for attributes sampling, and the even simpler Poisson distribution is often used in MUS. In this section, we show that these approximations are conservative in audit applications encountered in practice, even though one can construct theoretically possible scenarios where they would not be. Specifically:

- In sample design, Poisson-based sample sizes are ordinarily larger than binomial-based sample sizes, which in turn are ordinarily larger than the "exact" sample sizes indicated by the hypergeometric distribution.
- In sample evaluations, Poisson-based upper confidence limits are ordinarily larger than binomial-based limits, which in turn are ordinarily larger than the "exact" limits indicated by the hypergeometric distribution.

We show first that, for the parameters we care about in auditing, Poisson probabilities exceed binomial probabilities, which, in turn, exceed hypergeometric probabilities.

As before, we use the notation $CHyp(k,n,L,N)$, $CBin(k,n,p)$, and $CPoi(k,r)$ for the cumulative hypergeometric, binomial, and Poisson probabilities, respectively, that the number of errors does not exceed k given a sample size of n from a population of N items that contains L errors. Note that $p = L/N$ and $r = np$. If n_H is the "true" sample size calculated from the hypergeometric distribution, then n_B and n_P are the approximating sample sizes calculated using the binomial and Poisson distributions respectively.

Comparing the hypergeometric and binomial distributions, it can be shown that the binomial approximation of the hypergeometric overestimates the probability, that is,

$$CBin(k,n,p) > CHyp(k,n,L,N),$$

13

when

$$\frac{k}{n} \leq p\left(\frac{n-1}{n}\right)\left(\frac{N+1}{N}\right) - \left(\frac{n-1}{n}\right)\frac{1}{N} ; \tag{25}$$

and it underestimates the probability, that is,

$$CBin(k,n,p) < CHyp(k,n,L,N) ,$$

when

$$\frac{k}{n} \geq p\left(\frac{n-1}{n}\right)\left(\frac{N+1}{N}\right) \tag{26}$$

(Johnson, Kemp, and Kotz 2005, 270).

If n is chosen using the hypergeometric distribution, we can see from inequalities (25) and (26) that the hypergeometric probability will dominate the binomial only if k/n exceeds an amount slightly less than p. Because this is not likely in auditing, we may conclude that the condition in inequality (25) will be met in practice.

Comparing the binomial and Poisson distributions, it can be shown that the Poisson approximation of the binomial overestimates the probability, that is,

$$CPoi(k,r) > CBin(k,n,p)$$

when

$$\frac{k}{n} \leq p\frac{n}{n+1} ; \tag{27}$$

and it understates the probability, that is,

$$CPoi(k,r) < CBin(k,n,p) ,$$

when

$$\frac{k}{n} \geq p \tag{28}$$

(Johnson, Kemp, and Kotz 2005, 120).

If n is chosen using the binomial distribution, we can see from inequalities (27) and (28) that the binomial probability will dominate the Poisson only if k/n exceeds an amount slightly less than p. Clearly this is never the case in practice.

When the conditions in inequalities (25) and (27) are met, which they will be in auditing applications,

$$CPoi(k,r) > CBin(k,n,p) > CHyp(k,n,L,N) . \tag{29}$$

Figure 3, Conceptual Relationship Between Cumulative Distributions as Functions of *n*
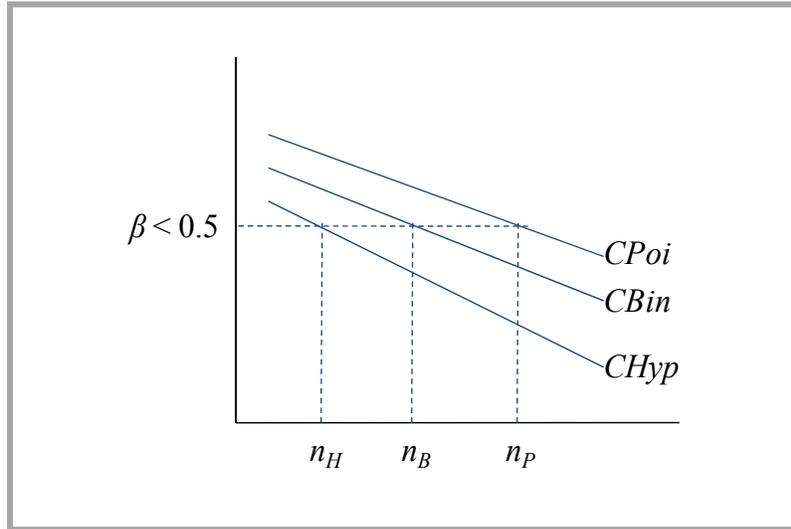
Because cumulative probability distributions are monotonically decreasing functions of *n* (the larger the sample the smaller the probability that only *k* or fewer errors will be selected), it therefore follows that for a desired risk of β, hypergeometric, binomial, and Poisson audit sample sizes are such that $n_H \leq n_B \leq n_P$. This is illustrated in figure 3.

Thus, when the condition in inequality (29) is met—which it will be in practice—the Poisson distribution formula yields sample sizes that are at least as large as those that are strictly necessary as indicated by the hypergeometric distribution.

Because cumulative probability distributions are monotonically decreasing functions of the total population error, a similar argument can be employed to demonstrate that, in practice, binomial and Poisson based upper confidence limits are increasingly conservative. This could be illustrated in a chart similar to figure 3 with upper limits along the *x*-axis rather than *n*.

Leslie et al. (1979) also comment on the conservatism of the Poisson relative to the binomial and that of the binomial relative to the hypergeometric.

2.5   Precision and Tolerable Misstatement in Classical Variables Sampling (Table D.1 of the Guide)

As explained in the guide (AAG-SAM 7.21) software for designing classical variables samples commonly requires the auditor to specify the α risk (of incorrect rejection) and the desired *allowance for risk of incorrect rejection*, also known as the *precision* of the classical variables estimate. Auditors, however, ordinarily want to set these parameters to also control for the risk about which they are typically more concerned, namely β risk (of incorrect acceptance) relative to tolerable misstatement. As explained in the guide, this can be done by setting precision at an appropriate fraction of tolerable misstatement related to α and β. The appropriate ratios of precision to tolerable misstatement for various α and β risks are tabulated in table D.1 of the guide. This section explains that table.

In classical variables sampling it is necessary to make an assumption about the distribution of the estimator. The central limit theorem assures us that the distribution will approach the normal distribution as sample size n increases; and the auditor's assumption is that n is large enough for the distribution to be approximately normal. Neter and Loebbecke (1975) studied the validity of this assumption empirically.

In what follows, the standard normal distribution (which has a mean of 0 and variance of 1) will be used, the cumulative version of which is

$$N(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} \exp(-t^2/2) dt \; . \tag{30}$$

It can be represented as the area under the standard normal density function curve to the left of z. If $N(z_\varepsilon) = 1 - \varepsilon$, then

$$z_\varepsilon = N^{-1}(1 - \varepsilon) \; . \tag{31}$$

If the recorded population Y_0 is actually correct, then α is the risk of incorrectly rejecting the hypothesis that $Y = Y_0$. It can be represented by the tail of a normal distribution centered on Y_0, as illustrated in figure 4. If σ_n is the standard deviation of the estimator given a sample size of n and the test is for overstatement, then the allowance for risk of incorrect rejection is

$$A = z_\alpha \sigma_n \; , \tag{32}$$

and the lower limit is $Y_c = Y_0 - A$. If, on the other hand, Y_0 overstates the true value by T, the amount of tolerable misstatement, then the estimator will be normally distributed about $Y_0 - T$, and β, the risk of incorrectly accepting the hypothesis, will be represented by the tail of that distribution to the right of the critical value $Y_c = (Y_0 - T) + z_\beta \sigma_n$. The critical value for controlling both risks is thus Y_c, where

$$\begin{aligned} Y_c &= (Y_0 - T) + z_\beta \sigma_n \\ Y_c &= Y_0 - z_\alpha \sigma_n \end{aligned} \; .$$

By subtraction,

$$\sigma_n = \frac{T}{z_\alpha + z_\beta} \; .$$

Therefore, from equation (32), the ratio of the desired precision of the estimate to tolerable misstatement is

$$\frac{A}{T} = \frac{z_\alpha}{z_\alpha + z_\beta} \; . \tag{33}$$

To be conservative, this ratio is rounded down because that results in the smaller value of A for a given T and thus in a potentially larger sample size. See also Roberts (1978, 46).

Figure 4, Controlling for Both *α* and *β* Risks in an Overstatement Test

$$\alpha = 0.10$$
$$\beta = 0.05$$
$$z_\alpha = 1.28$$
$$z_\beta = 1.64$$
$$\frac{A}{T} = \frac{z_\alpha}{z_\alpha + z_\beta} = 0.437$$

$$Y_c = (Y_0 - T) + z_\beta \sigma_n$$
$$Y_c = Y_0 - z_\alpha \sigma_n$$
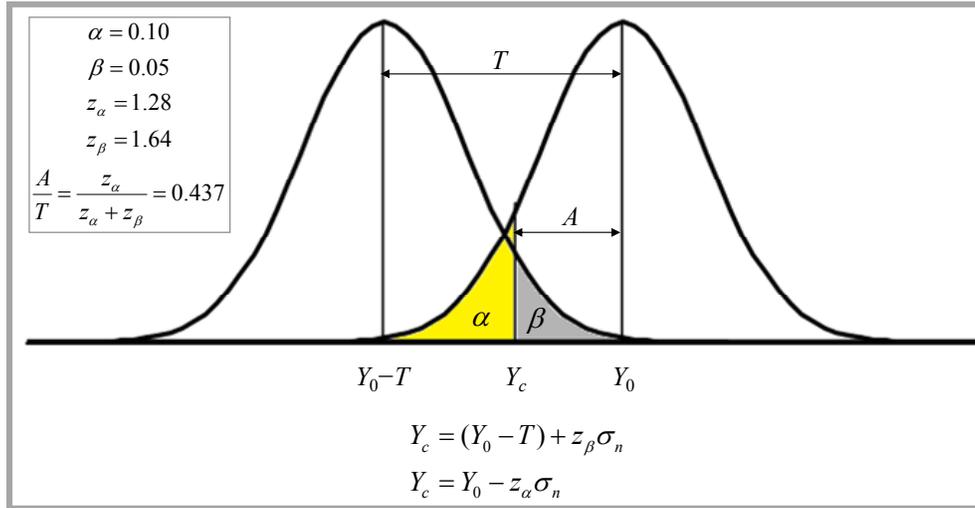
Example: See section 4.17 of this paper.

3 STATISTICAL FUNCTIONS IN EXCEL

Thanks to the statistical functions built into the software, all the computations described in this paper can be performed using Microsoft Excel (2007 and 2003 versions). Some are not easily done in a regular worksheet and are better programmed in VBA. Both built-in and illustrative VBA functions are described in this section.

3.1 Built-in Excel Functions

The following table shows the relationship between the functions described in the text and built-in Excel functions.

Statistical or Mathematical Function	Ref.	Excel Built-In Function
Hypergeometric mass function: $$Hyp(k,n,L,N) = \frac{\binom{L}{k}\binom{N-L}{n-k}}{\binom{N}{n}}$$	(2)	=HYPGEOMDIST(k, n, L, N)
Cumulative hypergeometric distribution: $$\beta = CHyp(k,n,L,N) = \sum_{i=k_0}^{k} Hyp(i,n,L,N)$$	(3)	Not available. Can be programmed in VBA by summing the built-in mass function.
Binomial mass function: $$Bin(k,n,p) = \binom{n}{k} p^k (1-p)^{n-k}$$	(5)	= BINOMDIST(k, n, p, FALSE)
Cumulative binomial distribution: $$\beta = CBin(k,n,p) = \sum_{i=0}^{k} Bin(i,n,p)$$	(6)	= BINOMDIST(k, n, p, TRUE)
Beta density function: $$b(p,1+k,n-k) = (n-k)\binom{n}{k} p^k (1-p)^{n-k-1}$$	(9)	Not available
Cumulative beta distribution: $$B(p,1+k,n-k) = \int_0^p b(t,1+k,n-k)dt$$	(10)	= BETADIST($p, 1+k, n-k$)
Inverse beta distribution: $$p = B^{-1}(1-\beta,1+k,n-k)$$	(12)	= BETAINV($1-\beta, 1+k, n-k$)
Poisson mass function: $$Poi(k,r) = \frac{r^k \exp(-r)}{k!}$$	(14)	= POISSON(k, r, FALSE)
Cumulative Poisson distribution: $$\beta = CPoi(k,r) = \sum_{i=0}^{k} Poi(i,r)$$	(15)	= POISSON(k, r, TRUE)

Statistical or Mathematical Function	Ref.	Excel Built-In Function
Gamma density function (standard): $g(r, 1+k) = \dfrac{r^k \exp(-r)}{\Gamma(1+k)}$	(16)	= GAMMADIST(r, $1+k$, 1, FALSE) The 3rd parameter is for scale: 1 for the *standard* gamma distribution.
Cumulative gamma distribution (standard): $1 - \beta = G(r, 1+k) = \int_0^r g(t, 1+k)dt$	(17)	= GAMMADIST(r, $1+k$, 1, TRUE)
Inverse gamma distribution: $r = G^{-1}(1-\beta, 1+k)$	(19)	= GAMMAINV($1-\beta$, $1+k$, 1)
Standard normal cumulative distribution: $1 - \varepsilon = N(z) = \dfrac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} \exp(-t^2/2)dt$	(30)	= NORMSDIST(z)
Inverse standard normal: $z_\varepsilon = N^{-1}(1-\varepsilon)$	(31)	= NORMSINV($1-\varepsilon$)
Round up to δ decimals: RoundUp(x, δ)		= ROUNDUP(x, δ)
Exponential: $\exp(x) = e^x$		= EXP(x)
Natural logarithm (logarithm to base e): $\ln(x)$		= LN(x) in worksheets, = LOG(x) in VBA code

3.2 Special-Purpose Functions in VBA

Several special-purpose VBA functions are mentioned in this paper. They are described in this section together with source code that can be copied into VBA modules and executed from a worksheet. While the implementations presented were developed with clarity and simplicity in mind rather than computational efficiency, the functions are also quite fast enough for most purposes.

3.2.1 Binomial Sample Sizes

Function (8), namely

$$n = \text{BinomSample}(\beta, p_E, p_T),$$

can be used to calculate attributes sample sizes using the binomial distribution. VBA program code is shown in figure 5. The program takes a simple "brute force" approach, starting off with a minimum value for n and then searching sequentially for a solution.

After checking the input parameters, the function computes the smallest possible value of n, which is the value of n if p_E and, hence, k_E were 0. From equation (6) this is the value of n such that

Figure 5, BinomSample VBA Function

```
Function BinomSample(risk As Double, pE As Double, pT As Double)
    Dim n As Double, k As Double
    If risk <= 0 Or risk >= 1 Or pE < 0 Or pE >= 1 Or pT <= 0 Or pT >= 1 Then
        BinomSample = CVErr(xlErrNum)
    Else
        n = Application.WorksheetFunction.RoundUp(Log(risk) / Log(1 - pT), 0)
        k = Application.WorksheetFunction.RoundUp(pE * n, 0)
        While Application.WorksheetFunction.BinomDist(k, n, pT, True) > risk And n <= 20000
            n = n + 1
            k = Application.WorksheetFunction.RoundUp(pE * n, 0)
        Wend
        BinomSample = IIf(Application.WorksheetFunction.BinomDist(k, n, pT, True) <= risk, n,
CVErr(xlErrNA))
    End If
End Function
```

$$CBin(0, n, p_T) = \binom{n}{0} p_T^0 (1 - p_T)^n = (1 - p_T)^n = \beta .$$

That is,

$$n = \frac{\ln(\beta)}{\ln(1 - p_T)} \tag{34}$$

rounded up to the nearest integer. Starting with this value the function searches sequentially for the first value of n that satisfies inequality (7),

$$CBin(k_E, n, p_T) \le \beta .$$

To avoid runaway computations the function is capped arbitrarily at $n = 20,000$, after which it returns a #N/A error.

3.2.2 *Poisson Sample Sizes*

Function (21), namely

$$n = \text{PoissonSample}(\beta, p_E, p_T) ,$$

can be used to calculate MUS sample sizes using the Poisson distribution. VBA program code is shown in figure 6. The program takes a simple "brute force" approach, starting off with a minimum value for n and then searching sequentially for a solution.

After checking the input parameters, the function computes the smallest possible value of n, which is the value of n if p_E and, hence, k_E were 0. From equation (15) and the fact that $r_T = np_T$,

Figure 6, PoissonSample VBA Function

```
Function PoissonSample(risk As Double, pE As Double, pT As Double)
    Dim n As Double
    If risk <= 0 Or risk >= 1 Or pE < 0 Or pE >= 1 Or pT <= 0 Or pT >= 1 Then
        PoissonSample = CVErr(xlErrNum)
    Else
        n = Application.WorksheetFunction.RoundUp(-Log(risk) / pT, 0)
        While Application.WorksheetFunction.GammaDist(n * pT, 1 + pE * n, 1, True) < 1 - risk And
n <= 20000
            n = n + 1
        Wend
        PoissonSample = IIf(Application.WorksheetFunction.GammaDist(n * pT, 1 + pE * n, 1, True)
>= 1 - risk, n, CVErr(xlErrNA))
    End If
End Function
```

$$\frac{(np_T)^0 \exp(-np_T)}{0!} = \beta \, ,$$

from which

$$n = \frac{-\ln(\beta)}{p_T} \tag{35}$$

rounded up to the nearest integer. Starting with this n, the function searches sequentially for the first value of n that satisfies inequality (20),

$$G(np_T, 1 + np_E) \geq 1 - \beta \, .$$

To avoid runaway computations, the function is capped arbitrarily at $n = 20,000$, after which it returns a #N/A error.

3.2.3   MUS Sample Design Factors

Function (23), namely

$$F = \text{MUSFactor}(\beta, e) \, ,$$

produces sample design factors used to compute sample sizes. VBA program code is shown in figure 7. It makes use of equation (22)

$$F = G^{-1}(1 - \beta, 1 + eF) \, . \tag{36}$$

After checking input parameters, the function starts by establishing an initial value of

$$F = G^{-1}(1 - \beta, 1) \, .$$

Figure 7, MUSFactor VBA Function

```
Function MUSFactor(risk As Double, e As Double)
    Dim F1 As Double, F As Double
    Dim i As Integer
    If risk <= 0 Or risk >= 1 Or e < 0 Or e >= 1 Then
        MUSFactor = CVErr(xlErrNum)
    Else
        F = Application.WorksheetFunction.GammaInv(1 - risk, 1, 1)
        If e = 0 Then
            MUSFactor = F
        Else
            F1 = 0
            i = 0
            While Abs(F1 - F) > 0.000001 And i <= 1000
                F1 = F
                F = Application.WorksheetFunction.GammaInv(1 - risk, 1 + e * F1, 1)
                i = i + 1
            Wend
            MUSFactor = IIf(Abs(F1 - F) <= 0.000001, F, CVErr(xlErrNum))
        End If
    End If
End Function
```

If $e = 0$, then that is the value; otherwise the initial F value feeds into the right-hand side of equation (36) to generate a new F. The process continues until successive values of F converge on the solution that satisfies equation (36). Sufficient convergence is deemed to have been achieved when the difference between two successive values of F is no greater than 1/1,000,000. To avoid runaway computations, the number of iterations is capped at 1,000, after which a #NUM! error is returned.

4   COMPUTATION OF TABLES WITH EXAMPLES

In this section, the tables in the guide are described and related to the preceding theoretical discussion. Examples are provided to demonstrate the calculations.

4.1   Table 3.1: Effect on Sample Size of Different Levels of Risk of Overreliance and Tolerable Deviation Rate

This table (AAG-SAM 3.41) illustrates the effect on sample size of the acceptable risk of incorrect acceptance, where the expected population error rate is zero.

Equation (34) is used to calculate *n* and the result is rounded up to the nearest integer. For example, given $\beta = 5\%$, $p_T = 10\%$, and $p_E = 0$,

$$n = \frac{\ln(\beta)}{\ln(1 - p_T)} = \frac{\ln(0.05)}{\ln(1 - 0.10)} = 28.4331\ldots,$$

rounded up to 29.

4.2   Table 3.2: Effect of Tolerable Rate on Sample Size

This table (AAG-SAM 3.52) illustrates the effect on sample size of the tolerable error rate under conditions where the expected population error rate is zero and the risk of incorrect acceptance is 10%.

Equation (34) is used to calculate *n*, which is then rounded up to the nearest integer. For example, given $\beta = 10\%$, $p_T = 3\%$, and $p_E = 0$,

$$n = \frac{\ln(\beta)}{\ln(1 - p_T)} = \frac{\ln(0.1)}{\ln(1 - 0.03)} = 75.5956\ldots,$$

rounded up to 76.

4.3   Table 3.3: Relative Effect of the Expected Population Deviation Rate on Sample Size

This table (AAG-SAM 3.56) illustrates the effect on sample size of the expected error rate, assuming the risk of incorrect acceptance is 5% and the tolerable error rate is 5%.

Function (8) is used to compute the sample sizes. For example, given $\beta = 5\%$, $p_T = 5\%$, and $p_E = 1.5\%$, the sample size is

$$n = \text{BinomSample}(0.05, 0.015, 0.05) = 124.$$

4.4 Table 3.4: Limited Effect of Population Size on Sample Size

This table (AAG-SAM 3.60) illustrates the effect of population size on sample size. The expected error rate is 1%, the tolerable error rate is 10%, and risk of incorrect acceptance is 10%.

Inequality (4) is used to determine the sample size. For example, suppose $\beta = 10\%$, $N = 100$, $p_T = 10\%$, and $p_E = 1\%$. Then, for $N = 100$, the tolerable error is $L_T = 100 \times 0.1 = 10$ (which would have been rounded up had it been necessary). From inequality (4), the required sample size is the minimum value of n for which

$$CHyp\left(k_E, n, 10, 100\right) \le 0.1,$$

where $k_E = \text{RoundUp}(0.01n, 0)$. By trial and error it can be determined that $n = 33$ is the smallest value of n for which

$$CHyp\left(1, n, 10, 100\right) \le 0.1. \tag{37}$$

To illustrate, for $n = 32$ to 34, we have

n	*Probability*
32	0.1079
33	0.0958
34	0.0849

from which it can be seen that the solution to inequality (37) is $n = 33$. The probabilities are obtained in Excel by summing the hypergeometric function from 0 to 1. For example, for $n = 33$, the probability is

$$\text{HYPGEOMDIST}(0,33,10,100) + \text{HYPGEOMDIST}(1,33,10,100) = 0.0958\ldots$$

Note that this illustrates a solution handcrafted for the specific example. A more general solution requires some programming.

4.5 Table 4.2: Table Relating RMM, Analytical Procedures (AP) Risk, and Test of Details (TD) Risk

The audit risk model is

$$RMM \times AP \times TD = AR.$$

Table 4.2 (AAG-SAM 4.40) shows the appropriate maximum *TD* risk given *RMM* and *AP*, and assuming *AR* = 5% Thus,

$$TD = \frac{AR}{RMM \times AP} = \frac{0.05}{RMM \times AP}.$$

For example, when *RMM* = 100% and *AP* = 30%,

$$TD = \frac{0.05}{1.00 \times 0.30} = 0.1666... ,$$

conservatively rounded down to 16% (rounding down results in a potentially larger sample size than not rounding).

4.6 Table 4.5: Illustrative Sample Sizes

This table (AAG-SAM 4.68) presents sample sizes for various levels of risk of incorrect acceptance, tolerable error rates, and expected error rates. For the purposes of this table, expected error rates are expressed as ratios of tolerable error rates.

Function (21) is used to compute the sample sizes. For example, given $\beta = 5\%$, $p_T = 4\%$, and a ratio of expected to tolerable of 0.30, we have $p_E = 0.30 \times 0.04 = 0.012$ and a required sample size of

$$n = \text{PoissonSample}(0.05, 0.012, 0.04) = 150.$$

The expected sum of taints in the rightmost column is $np_E = 150 \times 0.012 = 1.80$.

4.7 Table 4.6: Confidence (Reliability) Factors

The formula noted near the table in the guide (AAG-SAM 4.72) can be more succinctly written as

$$n = \frac{Factor}{p_T}.$$

The relationship between *Factor* and risk is given by the Poisson distribution in equation (35), namely $Factor = -\ln(\beta)$. Thus, for example, for $\beta = 0.14$, $Factor = -\ln(0.14) = 1.9661\ldots$, rounded up to 2.

4.8 Table 6.1: Confidence (Reliability) Factors

This table (AAG-SAM 6.23) duplicates table 4.6 (AAG-SAM 4.72). See section 4.7 of this paper.

4.9 Table 6.2: 5% of Incorrect Acceptance

The confidence factors in this table (AAG-SAM 6.36) are straight from table C.3 (AAG-SAM APP C, 144) and the incremental changes in factors are the increments. See section 4.16 of this paper.

4.10 Table A.1: Statistical Sample Sizes for Tests of Controls—5% Risk of Overreliance

This table (AAG-SAM APP A, 130) presents sample sizes for various tolerable and expected error rates given 5% acceptable risk of incorrect acceptance.

Function (8) is used to compute the sample sizes. The numbers in parentheses are the numbers of expected errors in the sample rounded up to the nearest integer. For example, given $\beta = 5\%$, $p_T = 6\%$, and $p_E = 2.5\%$, the sample size is

$$n = \text{BinomSample}(0.05, 0.025, 0.06) = 150 \,.$$

The expected number of errors in the sample is $k_E = 150 \times 0.025 = 3.75$, rounded up to 4. The complete entry is tabulated as "150 (4)."

4.11 Table A.2: Statistical Sample Sizes for Tests of Controls—10% Risk of Overreliance

This table (AAG-SAM APP A, 131) is the same as table A.1 (AAG-SAM APP A, 130), except the risk is 10% (see section 4.10 of this paper).

4.12 Table A.3: Statistical Sampling Results Evaluation Table for Tests of Controls—Upper Limits at 5% Risk of Overreliance

This table (AAG-SAM APP A, 132) presents upper error rate limits, given various sample sizes and numbers of selected errors, for a 5% risk of incorrect acceptance.

Equation (12) is used to calculate the upper error rate limits, which are then rounded up to the nearest tenth of a percentage point and displayed as a percentage. For example, given $\beta = 5\%$, $n = 50$, and $k = 2$, then from equation (12) the upper limit for 95% confidence is

$$p_U = B^{-1}(1 - 0.05, 1 + 2, 50 - 2) = 0.1206141\dots,$$

which is displayed as 12.1. In Excel, BETAINV(0.95,3,48) = 0.1206141...

4.13 Table A.4: Statistical Sampling Results Evaluation Table for Tests of Controls—Upper Limits at 10% Risk of Overreliance

This table (AAG-SAM APP A, 133) is the same as table A.3 (AAG-SAM APP A, 132), except the risk is 10% (see section 4.12 of this paper).

4.14 Table C.1: Monetary Unit Sample Size Determination Tables

This table (AAG-SAM APP C, 140) duplicates table 4.5 (AAG-SAM 4.68). See section 4.6 of this paper.

4.15 Table C.2: Confidence Factors for Monetary Unit Sample Size Design

Function (23) can be used to calculate the factors in this table (AAG-SAM APP C, 143). For example, if β = 5% and the ratio of expected to tolerable misstatement is e = 0.2, then

$$F = \text{MUSFactor}(0.05, 0.2) = 4.6201\ldots,$$

which is rounded up to 4.63. This can be used to derive sample sizes as in equation (24). For example, if $p_T = 0.03$, then $n = 4.63/0.03 = 154.333\ldots$, rounded up to 155. This agrees with the equivalent entry in table C.1 (see section 4.14 of this paper).

4.16 Table C.3: Monetary Unit Sampling−Confidence Factors for Sample Evaluation

This table (AAG-SAM APP C, 144) presents factors for calculating the upper error limit for overstatement errors for various levels of risk of incorrect acceptance and number of selected errors.

Function (19) is used to compute the r_U factors, which are rounded up to the nearest two decimals. For example, given β = 5% and k = 2, then from equation (19) the 95% upper limit is

$$r_U = G^{-1}(1 - 0.05, 1 + 2, 1) = 6.2957\ldots,$$

which is rounded up to 6.30. In Excel, GAMMAINV(0.95,3,1) = 6.2957...

These factors can be translated into monetary terms. Because r_U is the upper limit of the mean number of errors in samples of size n, the estimated upper limit of the mean error per dollar is r_U / n; and, therefore, if the population size is M dollars, the upper limit of monetary error is $M \times r_U / n$. In the example, if the population size is M = \$3,000,000 and the sample size is n = 300, then the monetary 95% upper error limit is $3,000,000 \times 6.30/300 = \$63,000$.

4.17 Table D.1: Ratio of Desired Allowance for Sampling Risk to Tolerable Misstatement

This table (AAG-SAM APP D, 147) gives the ratio A/T between the precision of the estimate A and tolerable misstatement T given the acceptable risks of incorrect rejection α and incorrect acceptance β. Both α and β are one-sided risks.

Equation (33) gives the ratio in terms of the standard normal deviates corresponding to α and β. For example, if α = 10% and β = 5%, then, as depicted in figure 4, $z_\alpha = N^{-1}(1 - 0.10) = 1.28155$ and $z_\beta = N^{-1}(1 - 0.05) = 1.64485$. Therefore

$$\frac{A}{T} = \frac{1.28155}{1.28155 + 1.64485} = 0.43792,$$

which is rounded down to 0.437 in table D.1. Rounding down is conservative in that it results in a smaller A for a given T and, therefore, a potentially larger sample. In Excel, NORMSINV(0.90) = 1.28155… and NORMSINV(0.95) = 1.64485…. This table also appears in Roberts (1978, 247).

4.18 Tables Not Described

Several tables in the guide are not described in this paper, for a variety of reasons. For completeness, they are listed in the following table.

Table	Title	Reason Not Discussed
3.5	Small Population Sample Size Table (AAG-SAM 3.62)	Nonstatistical
4.1	Example of Stratification (AAG-SAM 4.19)	Nonstatistical
4.3	Factors to Consider in Setting Tolerable Misstatement (AAG-SAM 4.53)	Nonnumeric
4.4	Factors Influencing Sample Sizes for a Substantive Test of Details in Sample Planning (AAG-SAM 4.63)	Nonnumeric
5.1	100 Percent Examination and Sample Testing Summary (AAG-SAM 5.12)	Nonstatistical
6.3	Calculation of Total Projected Misstatement	Evident from table
6.4	Calculating the Allowance for Sampling Risk	Evident from table
6.5	Andrews's Calculation of Projected Misstatement	Evident from table
B.1	Four Step Sequential Sampling Plan (AAG-SAM APP B, 136)	From Roberts (1978, 246)
B.2	Example of a Two-Stage Sampling Plan (AAG-SAM APP B, 138)	From *Montgomery's Auditing* (O'Reilly et al. 1999, 16:47)
C.4	Alternative MUS Sample Size Determination Using Expansion Factors (AAG-SAM APP C, 145)	Legacy table (as noted, new table C.2 can be more accurate)

REFERENCES

Abramowitz, Milton, and Irene A. Stegun. 1972. *Handbook of Mathematical Functions*. New York: Dover Publications.

DeGroot, Morris H., and Mark J. Schervish. 2002. *Probability and Statistics*, 3rd ed. Boston: Addison-Wesley.

Feller, William. 1968. *An Introduction to Probability Theory and Its Applications*, Vol. 1, 3rd ed. New York: John Wiley & Sons.

Freund, John E. 1972. *Mathematical Statistics*, 2nd ed. London: Prentice-Hall International.

Johnson, Norman L., Adrienne W. Kemp, and Samuel Kotz. 2005. *Univariate Discrete Distributions*, 3rd ed. New York: John Wiley & Sons.

Johnson, Norman L., Samuel Kotz, and N. Balakrishnan. 1994. *Continuous Univariate Distributions*, Vol. 1, 2nd ed. New York: John Wiley & Sons.

Leslie, Donald A., Albert D. Teitlebaum, and Rodney J. Anderson. 1979. *Dollar-Unit Sampling*. Toronto: Copp Clark Pitman.

Neter, John, and James K. Loebbecke. 1975. *Behavior of Major Statistical Estimators in Sampling Accounting Populations: An Empirical Study*. New York: American Institute of Certified Public Accountants.

O'Reilly, Vincent M., Patrick J. McDonnell, Barry N. Winograd, and James S. Gerson. 1999. *Montgomery's Auditing*, 12th ed. New York: Wiley.

Raiffa, Howard, and Robert Schlaifer. 2000. *Applied Statistical Decision Theory*. New York: Wiley Classics Library.

Roberts, Donald M. 1978. *Statistical Auditing*. New York: American Institute of Certified Public Accountants.